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Comments on spin operators and spin-polarization states of $2+1$ fermions

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Abstract. In this brief article we discuss spin-polarization operators and spin-polarization states of $2+1$ massive Dirac fermions and find a convenient representation by the help of 4-spinors for their description. We stress that in particular the use of such a representation allows us to introduce the conserved covariant spin operator in the $2+1$ field theory. Another advantage of this representation is related to the pseudoclassical limit of the theory. Indeed, quantization of the pseudoclassical model of a spinning particle in $2+1$ dimensions leads to the 4-spinor representation as the adequate realization of the operator algebra, where the corresponding operator of a first-class constraint, which cannot be gauged out by imposing the gauge condition, is just the covariant operator previously introduced in the quantum theory.

I

The $2+1$-spinor field theory [1] has attracted in recent years great attention due to various reasons, e.g., because of non-trivial topological properties, and due to the possibility of the existence of particles with fractional spins and exotic statistics (anyons), having probably applications to the fractional Hall effect, high-T_c superconductivity and so on [2]. In many practical situations the quantum behavior of spin 1/2 fermions (from now on simply called fermions) in $2+1$ dimensions can be described by the corresponding Dirac equation with an external electromagnetic field. The main difference between the relativistic quantum mechanics of fermions in $3+1$ and in $2+1$ dimensions is related to the different description of spin-polarization states. It is well known that in $3+1$ dimensions there exist two massive spin 1/2 fermions, the electron and its corresponding antiparticle, i.e., the positron. Both the electron and the positron have two spin-polarization states. In $2+1$ dimensions there exist four massive fermions: two different types of electrons and two corresponding positrons. In contrast to the situation in $3+1$ dimensions, each particle in $2+1$ dimensions has only one polarization state. We recall that constructing the covariant and conserved spin operators for the $3+1$ Dirac equation in an external field is an important problem as regards finding exact solutions of this equation and specifying the spin-polarization states [3]. Here, there do not exist universal covariant conserved spin operators which relate to any external field; for each specific configuration of the external field one has to determine such operators [4]. At first glance, this problem does not exist in $2+1$ dimensions, since each fermion has only one spin-polarization state. Nevertheless, the spin (or spin magnetic momentum) as a physical quantity in $2+1$ dimensions does exist, and therefore, the corresponding operators do exist. One can see, by solving the Dirac equation in $2+1$ dimensions, that knowledge of such spin operators is very useful for finding physically meaningful solutions. Moreover, it turns out that in $2+1$ dimensions the appropriate spin operator serves at the same time as a particle species operator whilst its explicit expression is useful for the interpretation of the theoretical constructions. In this brief article, we discuss spin-polarization operators and spin-polarization states of $2+1$ massive Dirac fermions and some convenient representations for their description.

II

It is well known that in $2+1$ dimensions (as well as in any odd number of dimensions) there exist two inequivalent sets (representations) of gamma matrices. In fact, the proper orthochronous Lorentz group $L_+^\uparrow$, in a pseudo-euclidean space $\mathcal{M}$, can be identified with the $SL(2,R)$ group of real unimodular 2×2 matrices, associated to linear transformations of unity determinant in a real two-dimensional vector space [5]. Considering the space $\mathcal{M}$ of real hermitian matrices spanned by the vector basis $\{\tau_\alpha\}$,

$$\tau_0 = \mathbf{1}\,, \quad \tau_1 = \sigma_3\,, \quad \tau_2 = \sigma_1\,,$$

where the $\sigma_{1,3}$ belong to the set of Pauli matrices $\sigma_i, i = 1,2,3$, one can associate to each vector in this space a matrix

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in $\mathcal{M}$ via the isomorphism

$$L_+^\uparrow \approx SL(2,R)/Z\,,$$

where Z denotes the center $\{I,-I\}$ of the $SL(2,R)$ group, which constitutes the (real) spinor representation of the Lorentz group.

An operator in $SL(2,R)$ can be represented by a matrix in the hermitian basis $\{\tau_\alpha\}$ or simply by a matrix formed by the product of any two elements of such a basis as, for example, the anti-hermitian matrix $\tau_3=\tau_1\tau_2$ in the $SL(2,R)$ associated algebra. The new (non-hermitian) basis $\{\tau_1,\tau_2,\tau_3\}$ of $SL(2,R)$ is also the set of generators of the real Clifford algebra [6]

$$[\tau_i,\tau_j]_+=2\overline{g}_{i\,j}\,;\quad i,j=1,2,3\ ,$$

where $\overline{g}$ is the metric tensor of a pseudo-euclidean space of signature $(+,+,-)$.

The complexifications

$$\Gamma_s^0=\tau_1=\sigma_3\,,\quad \Gamma_s^1=\tau_3=\mathrm{i}\sigma_2\,,\quad \Gamma_s^2=-s\mathrm{i}\tau_2=-s\mathrm{i}\sigma_1\,,\quad s=\pm1\ , \tag{1}$$

of the above Clifford algebra give rise to the algebra of different representations for Dirac gamma matrices, labeled by the subscript $s=\pm1$.

As a consequence, there exist, respectively, two different Dirac equations and two different Lagrangians for the corresponding spinor fields. If an external electromagnetic field is present, then the particle $(\zeta=1)$ and antiparticle $(\zeta=-1)$ with the charges $\zeta e,\ e>0$ respectively obey the Dirac equations in which the operator $\mathrm{i}\partial_\mu$ has to be replaced by $P_\mu=\mathrm{i}\partial_\mu-\zeta eA_\mu(x)$, where the $A_\mu(x)$ are electromagnetic potentials. Thus, in fact, in $2+1$ dimensions we have four massive fermions (let us call the two different types of fermions up and down particles) and respectively four types of solutions of the $2+1$ Dirac equation (2-spinors $\Psi^{(\zeta,s)}(x)$):

$$\left(\Gamma_s^\mu P_\mu-m\right)\Psi^{(\zeta,s)}(x)=0\,,\quad x=(x^\mu)\ ,\quad \mu=0,1,2\,,$$
$$P_\mu=\mathrm{i}\partial_\mu-\zeta eA_\mu(x)\ ,\quad s,\zeta=\pm1\,. \tag{2}$$

In such a picture (and in stationary external fields that do not violate the vacuum stability), the only physical states are those from the upper energy branch, and only such states can be used in second quantization [11].

III

In order to define a spin magnetic momentum of the $2+1$ massive fermions let us set the external field to be a uniform constant magnetic field. In $2+1$ dimensions, the magnetic field has only one component $F_{21}=-F_{12}=B=$ const. The sign of B defines the "direction" of the field, the positive B corresponds to the "up" direction whereas the negative B corresponds to the "down" direction. In such a background, (2) can be reduced to the stationary form

$$H^{(\zeta,s)}\Psi_n^{(\zeta,s)}(\mathbf{x})=\varepsilon_n^{(\zeta,s)}\Psi_n^{(\zeta,s)}(\mathbf{x})\ ,$$
$$H^{(\zeta,s)}=-\Gamma_s^0\Gamma_s^kP_k+\Gamma_s^0m\,,$$
$$\Psi^{(\zeta,s)}(x)=\exp\left(-\mathrm{i}\varepsilon^{(\zeta,s)}x^0\right)\Psi^{(\zeta,s)}(\mathbf{x})\ ,$$
$$\varepsilon_n^{(\zeta,s)}>0\,,\quad \mathbf{x}=\left(x^1,x^2\right)\ . \tag{3}$$

As usual, we pass to the squared equation through the ansatz

$$\Psi^{(\zeta,s)}(\mathbf{x})=\left[\Gamma_s^0\varepsilon+\Gamma_s^kP_k+m\right]\Phi^{(\zeta,s)}(\mathbf{x})\,, \tag{4}$$

to obtain the following equation:

$$\left[\varepsilon_n^2-D^{(\zeta,s)}\right]\Phi_n^{(\zeta,s)}(\mathbf{x})=0\,,$$
$$D^{(\zeta,s)}=m^2+\mathbf{P}^2+\frac{\mathrm{i}}{4}\zeta eF_{\mu\nu}\left[\Gamma_s^\mu,\Gamma_s^\nu\right]$$
$$=m^2+\mathbf{P}^2-s\zeta eB\sigma^3\,,$$
$$\mathbf{P}=\left(P^1,P^2\right)\ . \tag{5}$$

The 2-component spinors $\Phi_n^{(\zeta,s)}(\mathbf{x})$ may be chosen in the form $\Phi^{(\zeta,s)}(\mathbf{x})=f_n^{(\zeta,s)}(\mathbf{x})\,\upsilon$, where $f_n^{(\zeta,s)}(\mathbf{x})$ are some functions and υ some constant 2-component spinors that classify the spin-polarization states. We select υ to obey the equation $\sigma^3\upsilon=\upsilon$. One can see that selecting υ to be the eigenvector of σ^3 with the eigenvalue -1, we do not obtain new linearly independent spinors $\Psi_n^{(\zeta,s)}(\mathbf{x})$. This is a reflection of the well known fact (see e.g. [7]) that massive $2+1$ Dirac fermions have only one spin-polarization state. This reflects the fact that, in $2+1$ dimensions, the mass terms in the corresponding Lagrangians for the spinor fields $\Psi^{(\zeta,s)}$ are not invariant under a parity transformation, which consists in the inversion of one of the space coordinate axes, say, the x-axis.

In fact, under the transformation

$$\mathcal{P}\ :\quad \mathbf{x}\to\mathbf{x}'=(-x,y)\ ,$$

the spinor $\Psi^{(\zeta,+)}(x)$, which satisfies the above Dirac equation for fermions of mass m, transforms as

$$\Psi^{(\zeta,+)}(x)\to\Psi'^{(\zeta,-)}(x')=\mathcal{P}\Psi^{(\zeta,+)}(x)\ ,$$

where the components of the spinor $\Psi'^{(\zeta,-)}$ obey the equations

$$(P_0\Psi'^{(\zeta,-)}_1(x')+P_1\Psi'^{(\zeta,-)}_2(x')+\mathrm{i}P_2\Psi'^{(\zeta,-)}_2(x'))$$
$$=m\Psi'^{(\zeta,-)}_1(x')\,,$$
$$(-P_0\Psi'^{(\zeta,-)}_2(x')-P_1\Psi'^{(\zeta,-)}_1(x')+iP_2\Psi'^{(\zeta,+)}_1(x'))$$
$$=m\Psi'^{(\zeta,+)}_2(x')\,,$$

in any Lorentz reference frame. Hence, we can verify that for

$$\Psi'^{(\zeta,-)}_1(x')=\Psi_2^{(\zeta,+)}(x)\quad\text{and}\quad\Psi'^{(\zeta,-)}_2(x')=-\Psi_1^{(\zeta,+)}(x)\,,$$

i.e.,

$$\Psi'^{(\zeta,-)}(x')=\Gamma_+^1\Psi^{(\zeta,+)}(x)\ ,$$

we obtain a Dirac equation for a particle of mass $-m$. Therefore, we associate to the operator $\mathcal{P}$ which acts on $\Psi^{(\zeta,+)}(x)$ the gamma matrix Γ^1_+. We are thus led to the conclusion that in order to get new solutions of the Dirac equation from the corresponding $\Psi^{(\zeta,s)}(x)$ ones, besides an internal transformation such as the above parity transformation, it is necessary to perform the change $m \leftrightarrow -m$; these solutions must indeed be related to the solutions $\Psi^{(\zeta,-s)}(x)$, due to the existence of only two inequivalent representations of Dirac gamma matrices.

In a weak magnetic field it follows from (5) that

$$\varepsilon_n^{(\zeta,s)} = \varepsilon_n^{(\zeta,s)}\Big|_{B=0} - \mu^{(\zeta,s)} B\,,$$
$$\mu^{(\zeta,s)} = \frac{s\zeta e}{2\sqrt{m^2 + \left(f_n^{(\zeta,s)}\right)^{-1} \mathbf{P}^2 f_n^{(\zeta,s)}}}\,. \tag{6}$$

We have to interpret $\mu^{(\zeta,s)}$ as the spin magnetic momentum of $2+1$ fermions. Thus in $2+1$ dimensions, we have

$$\operatorname{sign} \mu^{(\zeta,s)} = s\zeta\,. \tag{7}$$

One ought to remark that this result matches with the conventional description of spin polarization in $2+1$ dimensions. Considering the total angular momentum in the rest frame (see, for example, [7,9]), one can define the operators $S_0^{(s)}$ of spin projection on the x^0-axis,

$$S_0^{(s)} = \frac{\mathrm{i}}{4}\left[\Gamma_s^1, \Gamma_s^2\right] = \frac{s}{2}\sigma^3\,. \tag{8}$$

In the non-relativistic limit we obtain from (4) and (6)

$$\mu^{(\zeta,s)} = \frac{s\zeta e}{2m}\,, \quad \Psi_n^{(\zeta,s)}(\mathbf{x}) = 2m\Phi_n^{(\zeta,s)}\,.$$

In such a limit the Dirac spinors $\Psi_n^{(\zeta,s)}(\mathbf{x})$ are eigenfunctions of the operators (8),

$$S_0^{(s)}\Psi_n^{(\zeta,s)}(\mathbf{x}) = \frac{s}{2}\Psi_n^{(\zeta,s)}(\mathbf{x})\,.$$

Thus, one can consider

$$M^{(\zeta,s)} = \frac{\zeta e}{m} S_0^{(s)} \tag{9}$$

as the spin magnetic momentum operator. However, the operators $S_0^{(s)}$ are not covariant and are not conserved in the external field. Below we represent a conserved and covariant spin operator for $2+1$ massive fermions.

IV

Let us use a 4-component spinor representation for the wave functions to describe particles in $2+1$ dimensions. Namely, let us introduce 4-component spinors of the form

$$\psi^{(\zeta,+1)}(x) = \begin{pmatrix} \Psi^{(\zeta,+1)}(x) \\ 0 \end{pmatrix},$$
$$\psi^{(\zeta,-1)}(x) = \begin{pmatrix} 0 \\ \sigma^1\Psi^{(\zeta,-1)}(x) \end{pmatrix}. \tag{10}$$

These 4-component spinors are representatives of 2-component spinors $\Psi^{(\zeta,+1)}(x)$ and $\Psi^{(\zeta,-1)}(x)$. At the same time it is convenient to use three 4×4 matrices γ^0, γ^1, and γ^2 taken from the following representation [8] of $3+1$ gamma matrices:

$$\gamma^0 = \begin{pmatrix} \Gamma^0_{+1} & 0 \\ 0 & -\Gamma^0_{-1} \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} \Gamma^1_{+1} & 0 \\ 0 & -\Gamma^1_{-1} \end{pmatrix},$$
$$\gamma^2 = \begin{pmatrix} \Gamma^2_{+1} & 0 \\ 0 & \Gamma^2_{-1} \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}. \tag{11}$$

In the new representation, the 4-component spinors (10) obey the Dirac equation of the following form:

$$\left(\gamma^\mu P_\mu - m\right)\psi(x) = 0\,, \quad P_\mu = \mathrm{i}\partial_\mu - \zeta e A_\mu(x)\,,$$
$$x = (x^\mu)\,, \quad \mu = 0,1,2\,. \tag{12}$$

In fact, this equation can be considered as a result of a partial dimensional reduction of the $3+1$ Dirac equation. Stationary solutions of (12) can be expressed via solutions $\Phi_n^{(\zeta,s)}(\mathbf{x})$ of (5) as follows:

$$\psi_n^{(\zeta,s)}(x) = \exp\left(-\mathrm{i}\varepsilon_n^{(\zeta,s)}x^0\right)\left[\gamma^0\varepsilon_n^{(\zeta,s)} + \gamma^k P_k + m\right]\varphi^{(\zeta,s)}(\mathbf{x})\,,$$
$$\mathbf{x} = \left(x^1, x^2\right)\,,$$
$$\varphi^{(\zeta,+1)} = \begin{pmatrix} \Phi^{(\zeta,+1)} \\ 0 \end{pmatrix}, \quad \varphi^{(\zeta,-1)} = \begin{pmatrix} 0 \\ \sigma^1\Phi^{(\zeta,-1)} \end{pmatrix},$$
$$\varepsilon_n^{(\zeta,s)} > 0\,, \tag{13}$$

whereas the energy spectrum is the same as for (5). One can easily see that the 4-spinors $\varphi^{(\zeta,s)}$ are eigenvectors of the operator Σ^3 with the eigenvalues s being the particle species:

$$\Sigma^3\varphi^{(\zeta,s)} = s\varphi^{(\zeta,s)}\,, \quad \Sigma^3 = \mathrm{i}\gamma^1\gamma^2 = \begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}. \tag{14}$$

The operator Σ^3 commutes with the squared Dirac equation. This fact allows us to find a spin integral of motion for the Dirac equation (12). Such an integral of motion reads

$$\Lambda = \frac{\mathcal{H}\Sigma^3 + \Sigma^3\mathcal{H}}{4m}\,, \quad \mathcal{H} = -\gamma^0\gamma^k P_k + \gamma^0 m\,. \tag{15}$$

In the case under consideration, we obtain

$$\Lambda\psi^{(\zeta,s)} = \frac{s}{2}\psi^{(\zeta,s)}\,, \quad \Lambda = \frac{1}{2}\gamma^0\Sigma^3 = \frac{1}{2}\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}. \tag{16}$$

V

Now we can consider $2+1$ QFT of the spinor field that obeys (12). Such a QFT can be obtained by a standard quantization of the corresponding Lagrangian. Here the field operators have the form

$$\hat{\psi}(x) = \begin{pmatrix} \hat{\Psi}_{+1}(x) \\ \sigma^1 \hat{\Psi}_{-1}(x) \end{pmatrix}, \quad (17)$$

where the 2-component operators $\hat{\Psi}_s(x)$ describe particles of the s-species. Decomposing the field (17) into the solutions (13), we obtain four types of creation and annihilation operators: $a_{s,n}$ and $a^+_{s,n}$ which are operators of particles ($\zeta = +1$) and $b_{s,n}$ and $b^+_{s,n}$ which are operators of antiparticles ($\zeta = -1$). Thus, in the QFT under consideration all the types of $2+1$ fermions appear on the same footing.

In the QFT one can define the second-quantized operator $\hat{\Lambda}$ that corresponds to the operator Λ of the field theory,

$$\hat{\Lambda} = \frac{e}{m} \int \hat{\psi}^\dagger \Lambda \hat{\psi} \mathrm{d}\mathbf{x} . \quad (18)$$

It is easily to verify that such an operator is a scalar under $2+1$ Lorentz transformations and is conserved in any external field. We call the operator $\hat{\Lambda}$ the spin magnetic polarization operator. One can easily see that this operator is expressed via charge operators $\hat{Q}_s$ of $2+1$ fermions as follows:

$$\hat{\Lambda} = \frac{1}{2m}\left(\hat{Q}_{+1} - \hat{Q}_{-1}\right), \quad (19)$$

where

$$\hat{Q}_s = \frac{e}{2} \int \left[\hat{\Psi}^\dagger_s, \hat{\Psi}_s\right] \mathrm{d}\mathbf{x} = e \sum_n \left(a^+_{s,n} a_{s,n} - b^+_{s,n} b_{s,n}\right) , \quad s = \pm 1 . \quad (20)$$

Remark that the eigenvalues of the operator $\hat{\Lambda}$ in the one-particle sector coincide with the spin magnetic momenta $\mu^{(\zeta,s)} = s\zeta e/2m$ of the $2+1$ fermions in the rest frame.

We stress that in particular the use of a spinor representation with more than 2-components allows us to introduce the conserved covariant spin operator in the $2+1$ field theory. There is another argument (which is related to the first quantization procedure) in favor of such representations, discussed below.

VI

It was demonstrated in [10] that relativistic quantum mechanics of all the massive $2+1$ fermions can be obtained in the course of first quantization of a corresponding pseudoclassical action where the particle species s is not fixed. General state vectors are 16-component columns. The states with a definite charge sign ζ can be described by 8-component columns ϕ_ζ. The operators of space coordinates $\hat{X}^k$ and momenta $\hat{\mathcal{P}}_k$ act on these columns as

$$\hat{X}^k = x^k \mathbf{I} , \quad \hat{\mathcal{P}}_k = \hat{p}_k \mathbf{I} , \quad \hat{p}_k = -\mathrm{i}\partial_k .$$

Here, $\mathbf{I}$ is the 8×8 unit matrix. Besides this, the spin degrees of freedom are related to the operators

$$\hat{\xi}^1 = \frac{\mathrm{i}}{2} \operatorname{antidiag}\left(\gamma^1, \gamma^1\right) , \quad \hat{\xi}^2 = \frac{\mathrm{i}}{2} \operatorname{diag}\left(\gamma^2, \gamma^2\right) .$$

The operator of a conserved first-class (ungauged) constraint has the form

$$\hat{t} = \hat{\theta} - \hat{S} , \quad \hat{\theta} = \operatorname{diag}(\Lambda, \Lambda) , \quad \hat{S} = 2\mathrm{i}\hat{\xi}^2\hat{\xi}^1 .$$

To fix the gauge at the quantum level, one imposes according to Dirac the condition $\hat{t}\phi_\zeta = 0$ on the physical state vectors. At the same time we choose ϕ_ζ to be eigenvectors of the matrix $\hat{\theta}$,

$$\hat{\theta}\phi_{\zeta,s} = \frac{s}{2}\phi_{\zeta,s} .$$

We see that in the first-quantized theory under consideration the operator $\hat{S}$ acts as the operator Λ in the quantum mechanics of item IV,

$$\hat{S}\phi_{\zeta,s} = \frac{s}{2}\phi_{\zeta,s} .$$

Thus, we can interpret the operator $\hat{S}$ as a spin operator.

Finally, there exists a relation between the representations of one-particle quantum states in terms of $\phi_{\zeta,s}$ and $\psi^{(\zeta,s)}$. Such a relation reads

$$\phi_{\zeta,+1}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi^{(\zeta,+1)}(x) \\ \gamma^0 \psi^{(\zeta,+1)}(x) \end{pmatrix} ,$$

$$\phi_{\zeta,-1}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi^{(\zeta,-1)}(x) \\ \gamma^0 \psi^{(\zeta,-1)}(x) \end{pmatrix} .$$

One can easily demonstrate that these two representations are physically equivalent.

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